

1. KIRCHHOFF'S RULES & SIGN TRAPS

CRITICAL

- **Junction Rule (KCL):** $\sum I_{\text{in}} = \sum I_{\text{out}}$. At any junction, label branch currents with arrows. If answer is negative, actual flow is opposite.
- **Loop Rule (KVL):** $\sum \Delta V = 0$ along any closed loop.
 - **Resistor R :** Traversing *WITH* current arrow $\Rightarrow -IR$ (drop). Traversing *AGAINST* arrow $\Rightarrow +IR$ (rise).
 - **EMF Source $\mathcal{E}$:** Traversing from $-$ (short) to $+$ (long) $\Rightarrow +\mathcal{E}$. From $+$ to $-$ $\Rightarrow -\mathcal{E}$ (independent of current direction!).
 - **Terminal Voltage V_T with internal r :**
 - Discharging (supplying power): $V_T = \mathcal{E} - Ir \leq \mathcal{E}$.
 - **Charging (driven backward into $+$):** $V_T = \mathcal{E} + Ir > \mathcal{E}$ (CQ 12!).

2. MASTER WORKED 2-LOOP / 2-BATTERY CIRCUIT

Setup: Left branch ($\mathcal{E}_1, R_1$), middle ($R_2, I_2 \downarrow$), right ($\mathcal{E}_2$ opposing, R_3).

1. **Junction:** $I_1 = I_2 + I_3 \Rightarrow I_2 = I_1 - I_3$.

2. **Loop 1 (Left, CW):** $+\mathcal{E}_1 - I_1 R_1 - I_2 R_2 = 0 \Rightarrow \mathcal{E}_1 - I_1(R_1 + R_2) + I_3 R_2 = 0$.

3. **Loop 2 (Right, CCW):** $+\mathcal{E}_2 - I_3 R_3 - I_2 R_2 = 0 \Rightarrow \mathcal{E}_2 + I_1 R_2 - I_3(R_2 + R_3) = 0$.

4. **Solve 2x2:** Substitute I_2 into loops; solve for I_1, I_3 .

• *Opposing batteries in 1 loop:* $I = \frac{\mathcal{E}_{\text{net}}}{R_{\text{tot}}}$. Larger EMF wins.

3. RESISTOR NETWORK SHORTCUTS & SYMMETRIES

• **Current Divider:** $I_1 = I_{\text{tot}} \left(\frac{R_2}{R_1 + R_2} \right)$. Smaller R takes larger I .

• **Bridge / Ladder:** Work inward! Parallel pair $R_D \parallel R_E = \frac{R}{2}$; in series with $R_C \Rightarrow 1.5R$; in parallel with $R_B \Rightarrow \frac{R(1.5R)}{2.5R} = 0.6R$; in series with $R_A \Rightarrow R_{\text{tot}} = 1.6R$ (PE 12).

• **Resistor Cube (12 equal R):** Body diagonal: $R_{\text{eq}} = \frac{5}{6}R$. Face diagonal: $\frac{3}{4}R$. Along one edge: $\frac{7}{12}R$.

4. RC CIRCUITS: CHARGING & DISCHARGING

• **Charging ($Q(0) = 0$):** $\tau = RC$. $I_0 = \mathcal{E}/R$, $Q_{\text{max}} = C\mathcal{E}$.

$$Q(t) = Q_{\text{max}}(1 - e^{-t/\tau}) \quad \bullet \quad I(t) = I_0 e^{-t/\tau}$$

$$V_C(t) = \mathcal{E}(1 - e^{-t/\tau}) \quad \bullet \quad V_R(t) = \mathcal{E}e^{-t/\tau}$$

At $t = 0$: Capacitor acts as a *short circuit* ($V_C = 0, I = \mathcal{E}/R$).

At $t \rightarrow \infty$: Capacitor acts as an *open circuit* ($I = 0, V_C = \mathcal{E}$).

• **Discharging ($Q(0) = Q_0$):** $Q(t) = Q_0 e^{-t/\tau}$, $I(t) = \frac{Q_0}{RC} e^{-t/\tau}$, $V_C(t) = V_0 e^{-t/\tau}$.

• **Key Timing Multipliers:**

- $t = \tau \Rightarrow 63.2\%$ charged (36.8% remaining on discharge).

- Half-life: $t_{1/2} = \tau \ln(2) \approx 0.693\tau$.

- Reaching fraction f of max: $t = -\tau \ln(1 - f)$. (90% $\Rightarrow 2.30\tau$; 99% $\Rightarrow 4.61\tau$).

- Discharge to fraction p : $t = \tau \ln(1/p)$ (PE 11: 3% $\Rightarrow t = \tau \ln(33.3) = 3.51\tau$).

• **Power Dissipation in R :** $P(t) = I^2 R = P_0 e^{-2t/\tau}$ (effective time constant is $\tau/2$; half-power at $t = \frac{1}{2}\tau \ln 2 \approx 0.347\tau$).

• **50% Energy Rule:** Battery outputs $W = C\mathcal{E}^2$. Capacitor stores $U = \frac{1}{2}C\mathcal{E}^2$. Resistor dissipates $E_R = \frac{1}{2}C\mathcal{E}^2$ (independent of RI).

5. REAL BATTERIES & POWER TRANSFER

• **Max Power Transfer:** Load R_L gets max power when $R_L = r$.

$P_{\text{max}} = \frac{\mathcal{E}^2}{4r}$. Circuit efficiency is exactly $\eta = 50\%$.

• **Extracting $\mathcal{E}$ and r from I vs R Graph (GW 10.1):**

$$I = \frac{\mathcal{E}}{R+r} \Rightarrow \mathcal{E} = I(R+r), R=0 \Rightarrow I_{\text{sc}} = \mathcal{E}/r, R=r \Rightarrow I = I_{\text{sc}}/2.$$

• **Voltmeter / Ammeter Loading:** High R ($R \gg \sqrt{R_V R_A}$) $\Rightarrow$ voltmeter across both ammeter and R minimizes error.

6. CONSTANTS & PARTICLE MECHANICS

NOT ON SHEET

• $m_p = 1.67 \times 10^{-27} \text{ kg}$ • $m_e = 9.11 \times 10^{-31} \text{ kg}$ • $e = 1.60 \times 10^{-19} \text{ C}$

• $k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ • $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$

• $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$. Speed from energy: $v = \sqrt{2K/m}$.

• **Coulomb 2-Body Energy:** $K_1 + K_2 + U = \text{const}$, $U = \frac{k_e q_1 q_2}{r}$.

Equal mass particles fired apart at v_0 (PE 14): $mv_0^2 - \frac{k_e q^2}{r_0} = -\frac{k_e q^2}{r_{\text{max}}}$.

Escape velocity: $K + U = 0 \implies v_{\text{esc}} = \sqrt{\frac{k_e q^2}{mr_0}}$.

Electron + Proton fired apart: $m_e v_e = m_p v_p \implies v_p = \frac{m_e}{m_p} v_e$.

• **Closest Approach** ($v \rightarrow 0$): $\frac{1}{2} m v_0^2 = \frac{k_e Z^2 e^2}{r_{\text{min}}} \implies d_{\text{surf}} = r_{\text{min}} - R_{\text{nucleus}}$.

• **Turning Points in $V(x)$:** $E_{\text{tot}} = qV(x_{\text{turn}}) \implies v_{\text{max}} = \sqrt{2E_{\text{tot}}/m}$ at $V = 0$.

7. CONTINUOUS CHARGE POTENTIAL INTEGRALS

• **Finite Uniform Rod on Axis (GW 7.5):** Length L , charge Q on $[-L/2, L/2]$:

$V(x) = \frac{kQ}{L} \ln \left(\frac{x+L/2}{x-L/2} \right)$ for $x > L/2$.

• **Ring of Radius R on Axis z :** $V(z) = \frac{kQ}{\sqrt{z^2 + R^2}}$; $E_z = \frac{kQz}{(z^2 + R^2)^{3/2}}$.

• **Disk of Radius R on Axis z :** $V(z) = \frac{k\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z)$.

• **Conducting Sphere (R):** Inside ($r \leq R$): $E = 0$, $V = \frac{kQ}{R} = \text{const}$. Outside ($r \geq R$): $E = \frac{kQ}{r^2}$, $V = \frac{kQ}{r}$. Surface potential is flat!

• **Two Spheres (R_1, R_2 dist D):** $V_a = k \left(\frac{q}{R_1} + \frac{q}{D-R_1} \right)$ (GW 7.4).

• **Field to Potential:** $\Delta V = -\int_{x_1}^{x_2} E(x) dx$. If $E(x) = ax^3 - b/x$, $\int = \frac{a}{4}(x_2^4 - x_1^4) - b \ln(x_2/x_1)$ (PE 7).

8. DIELECTRIC TRAPS & CAPACITOR SWITCHING

• **Dielectric Insertion: Always increases capacitance** ($C = \kappa C_0$).

• **Isolated ($Q = \text{const}$):** $V = V_0/\kappa \downarrow$, $E = E_0/\kappa \downarrow$, $U = U_0/\kappa \downarrow$. Stored energy drops $\implies$ fringing fields do positive work pulling slab in.

• **Connected to Battery ($V = \text{const}$):** $Q = \kappa Q_0 \uparrow$, $E = E_0$, $U = \kappa U_0 \uparrow$. Battery delivers $\Delta W = 2\Delta U$. Force pulls slab in: $F = +\frac{1}{2} V^2 \frac{d\kappa}{dx}$.

• **Dielectric Slicing Topologies:**

- Parallel boundary (series): $C_{\text{eff}} = \frac{2\epsilon_0\epsilon_1\epsilon_2}{\epsilon_1 + \epsilon_2} C_0$ (Harmonic mean).

- Perpendicular boundary (parallel): $C_{\text{eff}} = \frac{\epsilon_1 + \epsilon_2}{2} C_0$ (Arithmetic mean

$\rightarrow$ always higher C !).

• **Capacitor Charge Sharing / Switching (GW 8.4):**

Initial charge $Q_0 = C_1 V_0$. Connect in parallel with C_{load} :

$V_{\text{final}} = \frac{Q_0}{C_1 + C_{\text{load}}}$, $Q_1 = C_1 V_{\text{final}}$, $Q_{\text{load}} = C_{\text{load}} V_{\text{final}}$.

• **Series Voltage Divider:** $Q_1 = Q_2 \implies \frac{V_1}{C_1} = \frac{V_2}{C_2}$ (smaller cap drops larger V !).

9. CURRENT DENSITY & CONDUCTION MECHANICS

• **Volume Conservation (Wire Drawn/Stretched):** $\text{Vol} = AL = \text{const} \implies A = \text{Vol}/L$. Resistance: $R = \rho \frac{L}{A} \propto L^2 \propto \frac{1}{A} \propto \frac{1}{L}$.

- Double length ($L \rightarrow 2L$) $\implies R \rightarrow 4R$.

- Half resistance ($R \rightarrow R/2$) $\implies L \rightarrow L/\sqrt{2} \approx 0.707L$ (PE 10).

• **Non-Uniform Current Integration:** $I = \int_0^R J(r)(2\pi r dr)$.

If $J(r) = J_0(r/R) \implies I = \frac{2}{3} \pi R^2 J_0 \implies J_0 = \frac{3I}{2\pi R^2}$ (GW 9.4).

• **Time-Varying Current:** $I(t) = \frac{dQ}{dt}$. If $Q = Q_0(1 - e^{-t/\tau}) \implies I = \frac{Q_0}{\tau} e^{-t/\tau}$.

• **Drift vs Signal:** $v_d = \frac{J}{nq} \sim 10^{-4} \text{ m/s}$ (snail). Power transported by Poynting EM field wave $v_{\text{sig}} \sim 10^8 \text{ m/s}$.